Control of the TORA System through the IDA-PBC without Explicit Solution of Matching Equations

Pierluigi Arpenti, Diana Serra, Fabio Ruggiero, Vincenzo Lippiello
CREATE Consortium and Department of Electrical Engineering and Information Technology
University of Naples Federico II
Via Claudio 21, 80125, Naples, Italy
Email: {pierluigi.arpenti,diana.serra,fabio.ruggiero,vincenzo.lippiello}@unina.it

***Abstract*—This paper presents the control of a translational oscillator with a rotational actuator (TORA) system, in full gravity, through the interconnection and damping assignment passivity-based control (IDA-PBC). The sought goal is to control the underactuated TORA system while reducing the complexity in solving the partial differential equations coming out from the so-called matching equations, which arise from the IDA-PBC. The performance of the designed controller is illustrated through numerical simulations.**

I. Introduction

The port-Hamiltonian (pH) formalism is a particular representation of dynamic systems that explicitly reveal their physical properties concerning energy exchange, power flow, and interconnection structure. Such physical information is useful for nonlinear control design. Among the full range of control techniques available in the pH literature, the *interconnection and damping assignment passivity-based control* (IDA-PBC) [1] is here addressed. The IDA-PBC allows stabilizing a nonlinear system at the desired equilibrium by assigning an interconnection structure shaping the energy of the system such as to have the minimum at the desired equilibrium. Damping is then injected to ensure asymptotic stability, while the closed loop preserves the pH structure. The IDA-PBC is obtained by solving a set of partial differential equations (PDEs), the so-called *matching equations*, that arise from matching the open-loop and the desired closed-loop dynamics. Solving such PDEs is thus the bottleneck of the approach.

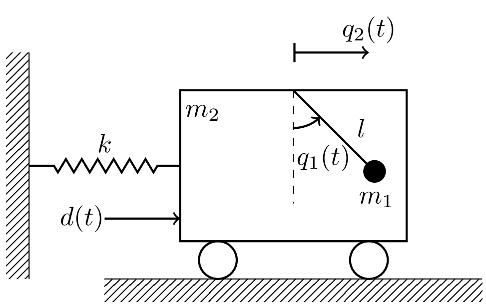


Fig. 1. Schematic representation of the TORA system in the vertical plane: $m_1 > 0$ and $m_2 > 0$ are the masses of the the rotational actuator and the horizontally oscillating platform, respectively; $l > 0$ is the length of the cable; $k > 0$ is the elasticity of the spring; $q_1(t)$ is the angular position of m_1 to the vertical; $q_2(t)$ is the horizontal displacement of m_2; $d(t)$ is an external disturbance applied on the platform. Notice that m_1 is an actuated rotating disk with radius $r > 0$ and inertia $I = m_1 r^2$.

In this paper, to reduce the computational effort, the approach proposed in [2] is employed. The aim pursued in [2] is to provide the exact solution for a subset of the PDEs, while transforming the complementary subset into algebraic equations. Such an approach can cope with both constant and configuration dependent mass matrix, that is separable and non-separable Hamiltonians, respectively. A systematic way to deal with underactuated planar systems is thus proposed in [2], with application to the rolling nonprehensile manipulation. In this paper, instead, the methodology is applied to stabilize the translational oscillator with a rotational actuator (TORA) system in full gravity. With reference to Fig. 1, the TORA is an underactuated mechanical system firstly introduced in [3], having a platform with mass m_2, constrained by a spring to oscillate in the horizontal plane, which is actuated through a rotational eccentric mass m_1, whose motion is exploited to damp the translational platform oscillation. The TORA system is often used as a benchmark and academic example to test several control designs by neglecting the gravity. In this paper, the gravity is duly taken into account. The novelty of the paper is the application of the procedure presented in [2] implemented for a mechanical system that is not a nonprehensile one, extending thus the validity of the approach.

In the literature, the model of the TORA system through the pH formalism and the use of the standard IDA-PBC approach is proposed in [4], where a constant closed-loop mass matrix reduces the complexity of the matching equations related to the kinetic energy. A dynamic extension is proposed in [5] to asymptotically stabilize the system with only position measurements. The procedure to address this goal is to shape the potential energy only, equating the open and the closed loop inertia matrices, canceling the assigned interconnection matrix to get rid of the kinetic energy PDEs, and then the resulting controller is independent of the velocity measurements because they are not present in the potential energy.

Besides, in the literature, the TORA system is used as an example to test different feedback-stabilizing controllers [6]. Several controllers based on cascade (linear cascade control, integrator backstepping) and passivity (feedback passivation, passivation without cancellation) paradigms can be used to asymptotically stabilize the system. The former class leads to algorithms requiring full state feedback linearization and nonlinearities cancellation, while the latter class, for input-output passive systems with relative degree one and weakly minimum-phase, leads to controllers with a reduced set of measurements and no cancellations. A Lagrangian-based change of coordinates along with a partial feedback linearization to reshape the system as a nonlinear cascade system in a strict feedback form is addressed in [7]. The global asymptotic stability is then achieved via a backstepping procedure. An experimental output regulation for the TORA system is performed in [8], while a piecewise multi-linear model is considered in [9]. Finally, fuzzy-based controls are proposed in [10] and [11].

II. IDA-PBC without Explicitly Solving the Matching Equations in a Nutshell

Consider an underactuated, undamped, planar, and mechanical system (i.e., with $n = 2$ state variables and $m = 1$ control input). Let $q = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^T \in \mathbb{R}^2$ be the vector of generalised coordinates, $p = \begin{bmatrix} p_1 & p_2 \end{bmatrix}^T \in \mathbb{R}^2$ the vector of generalised momenta, $u \in \mathbb{R}$ the control input, $I_2 \in \mathbb{R}^{2\times 2}$ the identity matrix, $O_2 \in \mathbb{R}^{2\times 2}$ the null matrix, $0_2 \in \mathbb{R}^2$ the zero vector, and $\xi = \begin{bmatrix} 1 & 0 \end{bmatrix}^T \in \mathbb{R}^2$ the input mapping term. The mathematical model of such a system can be represented through the following equations in the pH formalism

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} O_2 & I_2 \\ -I_2 & O_2 \end{bmatrix} \nabla H(q,p) + \begin{bmatrix} 0_2 \\ \xi \end{bmatrix} u, \tag{1}$$

where $H : \mathbb{R}^4 \to \mathbb{R}$ is the Hamiltonian function expressing the total energy (kinetic plus potential) stored in the system

$$H(q,p) = \frac{1}{2} p^T M^{-1}(q) p + V(q), \tag{2}$$

where $V(q) \in \mathbb{R}$ is the potential energy, and $M(q) \in \mathbb{R}^{2\times 2}$ is the positive-definite mass matrix having the following form

$$M(q) = \begin{bmatrix} b_{11}(q) & b_{12}(q) \\ b_{12}(q) & b_{22}(q) \end{bmatrix}. \tag{3}$$

The purpose of the IDA-PBC approach is to find a control law such that the closed-loop dynamics matches a target pH system with dissipation through the following equations

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} O_2 & M^{-1}(q) M_d(q) \\ -M_d(q) M^{-1}(q) & J_2(q,p) - \xi k_d \xi^T \end{bmatrix} \nabla H_d(q,p), \tag{4}$$

where $k_d > 0$ is a positive damping gain, $M_d(q) \in \mathbb{R}^{2\times 2}$ is the desired mass matrix that must be symmetric and positive definite (***Condition 1***), while $H_d : \mathbb{R}^4 \to \mathbb{R}$ is the following desired closed-loop Hamiltonian

$$H_d(q,p) = \frac{1}{2} p^T M_d^{-1}(q) p + V_d(q), \tag{5}$$

with $V_d(q) \in \mathbb{R}$ the desired potential energy such that $(q^*, 0_2) = \text{argmin } H_d(q,p)$ (***Condition 2***), and $J_2(q,p) \in \mathbb{R}^{2\times 2}$ is the assigned interconnection skew-symmetirc matrix (***Condition 3***).

The IDA-PBC approach designs the control input u such as to bring the open loop system (1) into the closed loop (4). Equating (1) with (4) yields

$$\begin{aligned} &\begin{bmatrix} O_2 & I_2 \\ -I_2 & O_2 \end{bmatrix} \nabla H(q,p) + \begin{bmatrix} O_{2x1} \\ \xi \end{bmatrix} u \\ &= \begin{bmatrix} O_2 & M^{-1}(q) M_d(q) \\ -M_d(q) M^{-1}(q) & J_2(q,p) - \xi k_d \xi^T \end{bmatrix} \nabla H_d(q,p). \end{aligned} \tag{6}$$

The control input can be designed from (6) provided that the following equation holds

$$\begin{aligned} \xi^{\perp} \big(\nabla_q H(q,p) - M_d(q) M^{-1}(q) \nabla_q H_d(q,p) \\ + J_2(q,p) M_d^{-1}(q) p \big) = 0 \end{aligned} \tag{7}$$

that is obtained by multiplying both sides of (6) by $\xi^{\perp}$, which is the left annihilator of ξ, and neglecting the damping term $-\xi k_d \xi^T$ that is included later. The expression in (7) constitutes a set of PDEs referred to as *matching equations*, and that can be split into the following two subsets

$$\begin{aligned} &\xi^{\perp} (\nabla_q (p^T M^{-1}(q) p) \\ &\quad - M_d(q) M^{-1}(q) \nabla_q (p^T M_d^{-1}(q) p) \\ &\quad + 2 J_2(q,p) M_d^{-1}(q) p) = 0 \end{aligned} \tag{8}$$

and

$$\xi^{\perp} (\nabla_q V(q) - M_d(q) M^{-1}(q) \nabla_q V_q) = 0 \tag{9}$$

referred to as *kinetic energy* and *potential energy matching equations*, respectively. Such matching equations must be solved for $M_d(q)$, $V_d(q)$, and $J_2(q,p)$ satisfying Conditions 1, 2, and 3. The energy shaping control law can be written as $u_{es} = (\xi^T \xi)^{-1} \xi^T (\nabla_q H(q,p) - M_d(q) M^{-1}(q) \nabla_q H_d(q,p) + J_2(q,p) M_d^{-1}(q) p)$, stabilizing the closed-loop dynamics at the desired equilibrium $(q,p) = (q^\star, 0_2)$ thanks to the choice of $M_d(q)$, $V_d(q)$, and $J_2(q,p)$ as mentioned above. Moreover, in order to guarantee asymptotic stability of the equilibrium point, it is possible to inject a *damping term* expressed as $u_{di} = -k_d \xi^T \nabla_p H_d(q,p)$. The sought IDA-PBC law is $u = u_{es} + u_{di}$ which, as expected, assigns the desired target dynamic (4) to the system (1). Further details can be found in [12].

The bottleneck of the described methodology consists in solving the PDEs (8) and (9). In order to avoid the explicity resolution of the the matching equations, the approach in [2] can be pursued. After computing the determinant $\Delta = b_{11}(q) b_{22}(q) - b_{12}^2(q) > 0$ of $M(q)$, the desired mass matrix can be parameterized as

$$M_d(q) = \Delta \begin{bmatrix} a_{11}(q,c_1) & a_{12}(q,c_1) \\ a_{12}(q,c_1) & a_{22}(q,c_1) \end{bmatrix} \tag{10}$$

where $c_1 \in \mathbb{R}^{n_{c1}}$, with $n_{c1} \geq 0$, is a set of gains useful to design the controller. Under this parameterization, the potential energy matching equation (9) can be transformed into

$$\nabla_{q_2} V(q) + \alpha(q,c_1) \nabla_{q_1} V_d(q,c_2) + \beta(q,c_1) \nabla_{q_2} V_d(q,c_2) = 0 \tag{11}$$

where $c_2 \in \mathbb{R}^{n_{c2}}$, with $n_{c2} \geq 0$, is an additional set of gains useful to design the controller, while $\alpha(q, c_1)$ and $\beta(q, c_1)$ are two scalar functions, defined as linear combination of the elements of $M(q)$ and $M_d(q)$ matrices [2]. Once these $\alpha(q, c_1)$ and $\beta(q, c_1)$ functions are properly designed, the desired potential energy $V_d(q, c_2)$ can be computed without explicitly solving (11) [2]. The gains c_1 and c_2 are set to comply with Condition 2: in case it is not possible to find any value for c_1 and c_2, the functions $\alpha(q, c_1)$ and $\beta(q, c_1)$ must be re-designed. Once $\alpha(q, c_1)$, $\beta(q, c_1)$, $V_d(q, c_2)$, c_1, and c_2 are correctly retrieved, it is possible to evaluate the following terms of $M_d(q)$ as

$$\begin{aligned} a_{12}(q, c_1) &= -\frac{\alpha(q, c_1) b_{11}(q) + \beta(q, c_1) b_{12}(q)}{\Delta} \\ a_{22}(q, c_1) &= -\frac{\alpha(q, c_1) b_{12}(q) + \beta(q, c_1) b_{22}(q)}{\Delta}. \end{aligned} \tag{12}$$

Through this choice, the desired closed loop mass matrix $M_d(q)$ is structurally symmetric, but to fully comply with Condition 1 it must be definite positive. Hence, according to the Sylvester's criterion, the conditions $a_{11} > 0$ and $\Delta_d > 0$ must be imposed, with Δ_d the determinant of M_d. Therefore, the conditions are satisfied if

$$a_{11}(q, c_1) = \frac{k_a a_{12}^2(q, c_1)}{a_{22}(q, c_1)} > 0 \tag{13}$$

where $k_a > 1$ is a constant parameter. Equivalently, the criterion is satisfied if

$$\alpha(q, c_1) b_{12}(q) + \beta(q, c_1) b_{22}(q) < 0, \tag{14}$$

in which c_1 must be designed without destroying the condition on the minimization of the desired potential energy. If this is not possible, it is necessary to re-design $\alpha(q, c_1)$ and $\beta(q, c_1)$ and find another solution until the conditions are met. Once c_1 is suitably find to satisfy (14), the terms of $M_d(q)$ can be in turn computed as in (12) and (13).

The matrix $J_2(q, p)$ can finally be employed to satisfy the kinetic matching equation (8). Following the approach in [13], to fulfil Condition 3, the interconnection matrix can be parameterized as

$$J_2 = \begin{bmatrix} 0 & j_2(q,p) \\ -j_2(q,p) & 0 \end{bmatrix}, \tag{15}$$

with $j_2(q, p) : \mathbb{R}^4 \to \mathbb{R}$ a scalar function. In such a way, the PDE (8) is transformed into the following algebraic equation

$$\begin{aligned} &\xi^{\perp} \nabla_q \left(p^T M^{-1}(q) p\right) \\ &\quad - \xi^{\perp} M_d(q) M^{-1}(q) \nabla_q \left(p^T M_d^{-1}(q) p\right) \\ &\quad - 2 j_2(q, p) \xi^T M_d^{-1} p = 0, \end{aligned} \tag{16}$$

whose solution is given by

$$\begin{aligned} j_2(q, p) = &\left(2\xi^T M_d^{-1}(q) p\right)^{-1} \left(\xi^{\perp} \nabla_q \left(p^T M^{-1}(q) p\right)\right. \\ &\left. - \xi^{\perp} M_d(q) M^{-1}(q) \nabla_q \left(p^T M_D^{-1}(q) p\right)\right). \end{aligned} \tag{17}$$

In the end, the control law u can be designed as before.

III. Mathematical Model and Control Design of the TORA System

The TORA is an underactuated, undamped, planar and mechanical system fitting the class of systems addressed in the previous section. The TORA is schematically represented and described in Fig. 1 and its caption. It consists of a mass constrained to move on a horizontal line, whose oscillations are damped by a rotation actuator mass.

As derived in [4], the components of the inertia matrix $M(q)$ in (3) for the TORA system are $b_{11} = m_1 l^2 + I$, $b_{12}(q_1) = m_1 l \cos(q_1)$, and $b_{22} = m_1 + m_2$, while the potential energy associated to the TORA system is $V(q_1, q_2) = \frac{1}{2} k q_2^2 + m_1 g l cos(q_1)$. Therefore, the total energy of the system is given by the Hamiltonian (2)

$$H(q, p) = \frac{1}{2} p^T M^{-1}(q_1) p + \frac{1}{2} k q_2^2 + m_1 g l cos(q_1). \tag{18}$$

Finally, the explicit dynamic model for the considered TORA system, expressed in the PH formalism, is obtained from (1) by including the external disturbance effect as

$$\dot{q_1} = \frac{b_{22} p_1 - p_2 b_{12}(q_1)}{\Delta}, \tag{19a}$$

$$\dot{q_2} = \frac{b_{11} p_2 - p_1 b_{12}(q_1)}{\Delta}, \tag{19b}$$

$$\begin{aligned} \dot{p_1} = &\frac{m_1 l (b_{11} b_{22} (b_{11} b_{22} g - p_1 p_2)) \sin(q_1)}{\Delta^2} \\ &+ \frac{m_1 l (m_1 (b_{22} p_1^2 + b_{11} p_2^2) l \cos(q_1)) \sin(q_1)}{\Delta^2} \\ &- \frac{m_1 l (m_1^2 (2 b_{11} b_{22} g - p_1 p_2) l^2 \cos^2(q_1)) \sin(q_1)}{\Delta^2} \\ &+ \frac{m_1 l (g m_1^4 l^4 \cos^4(q_1)) \sin(q_1)}{\Delta^2} + u, \end{aligned} \tag{19c}$$

$$\dot{p_2} = -k q_2 + d, \tag{19d}$$

with $\Delta = b_{11} b_{22} - b_{12}^2(q_1)$, the determinant of $M(q_1)$.

In order to design the control law, the external disturbance $d(t)$ is set to zero, and it will be added during the numerical tests for robustness analysis.

The sought goal is the stabilization of the system at the equilibrium $q^* = (0, 0)$. For this design, the scalar functions can be chosen as $\alpha(q, c_1) = \frac{c_1}{(q_1 l)^2 + q_2^2 + 1}$ and $\beta(q, c_1) = -\frac{1}{(q_1 l)^2 + q_2^2 + 1}$, with $n_{c1} = 1$. Notice that, as outlined in [2], such a particular design of the scalar functions α and β must be performed to guarantee the respect of the Conditions 1, 2 and 3. The existence of an explicit solution of (11) is guaranteed by choosing some particular, yet general, forms for the scalar functions α and β (see Appendix I of [2]). Replacing the selected scalar functions in (11), the potential energy matching equation has the following solution $V_d(q, c_2) = -\frac{k q_1 (3 q_1^3 + 12 c_1 q_1^2 q_2 + 4 c_1^3 q_2 (3 + 3 q_2^2 + q_1^2 l^2))}{12 c_1^4} - \frac{k q_1 (c_1^2 q_1 (6 + 18 q_2^2 + q_1^2 l^2))}{12 c_1^4} - 12 c_1^4 f\left(\frac{q_1 + c_1 q_2}{c_1}, c_2\right)$, where $f(\cdot, c_2) \in \mathbb{R}$ is a generic function of its argument, and $n_{c2} = 1$. Such a function $f(\cdot, c_2)$ must be chosen to comply with Condition 2, that is $V_d(q, c_2)$ must have a minimum at the target equilibria. To this aim, the function can be chosen as $f(\cdot, c_2) = \frac{\cos\left(\frac{c_2(q_1+c_1q_2)}{c_1}\right)}{12c_1^4}$. Through this choice, the gradient vector of the target potential energy is given by

$$\nabla_q V_d(q, c_2) = \begin{bmatrix} \frac{-k(3q_1^3+9c_1q_1^2q_2+c_1^2q_1(\psi)+3c_1^3(q_2+q_2^3+q_1^2q_2l^2))+3c_1^3c_2\sin(\phi)}{3c_1^4} \\ \frac{-kq_1(3q_1^2+9c_1q_1q_2+c_1^2(\psi))+3c_1^3c_2\sin(\phi)}{3c_1^3} \end{bmatrix} \quad (20)$$

with $\phi = \frac{c_2(q_1+c_1q_2)}{c_1}$, and $\psi = 3 + 9q_2^2 + q_1^2l^2$, whose value evaluated at q^* is $\nabla_q V_d(q)\Big|_{q^*} = 0_2$, qualifying q^* as a stationary point of $V_d(q, c_2)$. At the same time, the Hessian matrix related to the desired potential energy function evaluated at the desired equilibrium $q^\star$ is

$$\nabla_q^2 V_d(q)\Big|_{q^*} = \begin{bmatrix} \frac{-k+c_2^2}{c_1^2} & \frac{-k+c_2^2}{c_1} \\ \frac{-k+c_2^2}{c_1} & c_2^2 \end{bmatrix}, \quad (21)$$

that is positive definite if $c_1 > 0$ and $c_2 > \sqrt{k}$. Condition 2 is thus fulfilled.

Proceeding with the design, the problem related to the inequality (14) must be addressed as follows

$$\frac{-b_{22} + c_1 m_1 l \cos(q_1)}{1 + q_2^2 + q_1^2 l^2} < 0. \quad (22)$$

Inequality (22) is satisfied if $c_1 < \frac{b_{22}}{m_1 l}$, that is not in contrast with $c_1 > 0$ because $b_{22} > 0$. Therefore, suitable values for the gain are $c_1 \in \left]0, \frac{b_{22}}{m_1 l}\right[$.

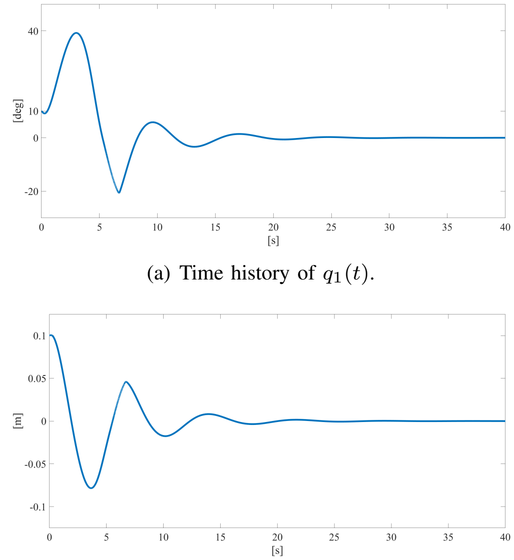
Furthermore, it is possible to compute the target mass matrix $M_d(q)$ from (12) and (13) as

$$M_d(q) = \begin{bmatrix} \frac{-c_3(b_{11}c_1-m_1l\cos(q_1))^2}{(1+q_2^2+q_1^2l^2)(-b_{22}+c_1m_1l\cos(q_1))} & \frac{-b_{11}c_1+m_1l\cos(q_1)}{1+q_2^2+q_1^2l^2} \\ \frac{-b_{11}c_1+m_1l\cos(q_1)}{1+q_2^2+q_1^2l^2} & \frac{b_{22}-c_1m_1l\cos(q_1)}{1+q_2^2+q_1^2l^2} \end{bmatrix}. \quad (23)$$

Choosing the skew-symmetric interconnection matrix as in (15), it is thus possible to solve the algebraic equation (16) for the scalar function $j_2(q, p)$ whose expression is given in (17) and not reported here for brevity. Hence, the energy shaping and the damping control terms can be computed as in the previous section.

IV. Simulations

Numerical simulations are carried out to evaluate the performance of the designed controller. The model parameters employed to simulate the TORA are taken from [14], and they are listed in the following $m_1 = 1$ kg, $m_2 = 10$ kg, $r = 0.1$ m, $l = 1$ m, $k = 5$ and $g = 9.81$ m/s^2. Where not otherwise specified, the IDA-PBC control law is designed upon these values. The sought goal is to stabilize the TORA system at the equilibrium point $q^\star = (0, 0)$ with zero generalised momenta. The simulations are performed on standard PC through the



(a) Time history of $q_1(t)$.

(b) Time history of $q_2(t)$.

Fig. 2. Case Study I. Test carried out with nominal conditions. The objective is to stabilize the TORA system at $q^\star = (0, 0)$.

MATLAB/Simulink environment. The considered simulation time is 40 s for all the case studies.

Two case studies are considered in the following. In the former case study, the controller is evaluated with nominal conditions, and no external disturbance, to stabilize the TORA system at $q^\star$. The second case study presents the results considering the stabilization of the system at $q^\star$ by introducing parametric uncertainties, noisy measurements, and a time delay introduced by the discretization of the controller, plus an external disturbance term $d(t)$.

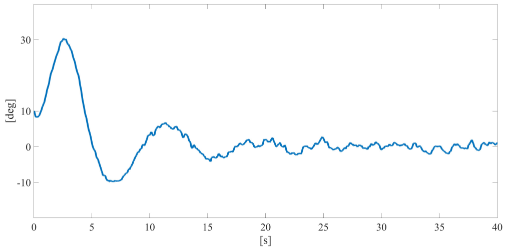
A. Case Study I

The first case study is carried out without uncertainties of any type and $d(t) = 0$. The controller is thus evaluated during nominal operating conditions, while its gains are experimentally tuned as $c_1 = 6$, $c_2 = 3$, $k_a = 2$, $k_v = 100$, which are suitable values concerning the discussion in Section III. The chosen initial conditions for the test are $q_1(0) = 10$ deg, $q_2(0) = 0.1$ m, $\dot{q}_1(0) = 0$ rad/s, and $\dot{q}_2(0) = 0$ m/s.

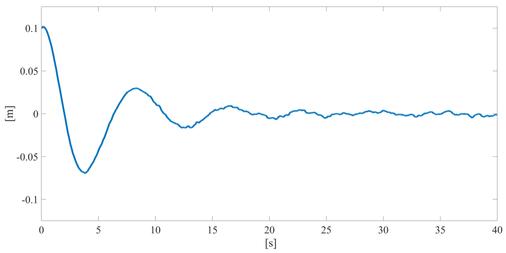
Fig.2 displays the time histories of $q_1(t)$, $q_2(t)$. The plots show that the designed controller forces the state trajectories to the desired equilibrium in a smooth fashion.

B. Case Study II

In order to verify the robustness of the controller, the following perturbations are introduced. Concerning the robustness to parametric uncertainties, the values of the model parameters employed within the IDA-PBC law are incremented as follows: 10 % for m_1,m_2,l and r, and 20 % for k. The parameters to simulate the TORA dynamics (18) are kept as the nominal values. Concerning the robustness to noisy measurements, a white noise is added to the signals $q_1(t)$, $q_2(t)$, $\dot{q}_1(t)$ and $\dot{q}_2(t)$, with a variance of 0.05, 0.01, 0.05, and 0.01,



(a) Time history of $q_1(t)$.



(b) Time history of $q_2(t)$.

Fig. 3. Case Study II. Test carried out by supposing parametric uncertainties, noisy measurements, discretization of the controller and a sinusoidal external disturbance. The objective is to stabilize the TORA system at $q^\star = (0, 0)$.

respectively. Concerning the robustness to a possible controller discretization, the output of the control law u is fed to the TORA dynamics with a sample time of $T_s = 0.01$ s, while the TORA dynamic equations (18) are solved in simulation through the *ode45* Matlab solver with a maximum step time of $0.1T_s$. Concerning the robustness to possible exogenous disturbance, $d(t) = 0.01 \sin(t)$ is introduced to further stress the robustness analysis of the controller.

The initial conditions, the sought stabilization goal at $q^\star$ and the controller gains are kept as the Case Study I. Fig.3 displays the time histories of $q_1(t)$, $q_2(t)$ and it shows that, besides to guarantee the convergence to the desired configuration and good robustness to model and measurement uncertainties, the controller weakens all the disturbance. As evidenced by Fig.3(b), the amplitude of the residual disturbance is indeed reduced of about one order of magnitude.

Remark. A comparison with state-of-art controllers can be in principle carried out. The controller designed in [14] was chosen as a basis for comparison. The results are similar with the ones previously obtained. Nevertheless, the sequence explained in [14] (partial feedback linearization plus backstepping) cannot be applied to all the underactuated, undamped, planar, and mechanical system tackled instead by the methodology presented in Section II. This assessment is clearly stated in [14]: an example is the ball-and-beam benchmark system that cannot be feedback linearized, while the approach presented here can handle it [2].

V. Conclusion and Future Work

An application to the TORA system of the IDA-PBC design without explicitly solving the matching equations was presented in this paper. The methodology leverages on a mass matrix parameterization that simplifies the solution of the potential energy matching equations, while the kinetic energy matching equations are transformed into algebraic equations. The obtained controller has been thus used to stabilize the considered benchmarking system in the desired equilibrium point. Robustness analysis was carried out in numerical tests by adding external disturbances, controller discretization, parametric uncertainties, and noisy measurements. Future work involves the addition of integral action to the IDA-PBC and the extension of the methodology to non-planar systems.

Acknowledgment

The research leading to these results has been supported by the RoDyMan project, which has received funding from the European Research Council FP7 Ideas under Advanced Grant agreement number 320992. The authors are solely responsible for the content of this manuscript.

References

[1] R. Ortega, A. Van Der Schaft, B. Maschke, and G. Escobar, "Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems," *Automatica*, vol. 38, no. 4, pp. 585–596, 2002.

[2] D. Serra, F. Ruggiero, A. Donaire, L. Buonocore, V. Lippiello, and B. Siciliano, "Control of nonprehensil planar rolling manipulation: A passivity based approach," *IEEE Transactions on Robotics*, 2018, DOI: 10.1109/TRO.2018.2887356.

[3] C.-J. Wan, D. Bernstein, and V. Coppola, "Global stabilization of the oscillating eccenteric rotor," *Nonlinear Dynamics*, vol. 10, pp. 49–62, 1996.

[4] A. Morillo, M. Bolivar, and V. Acosta, "Feedback stabilization of the TORA system via interconnection and damping assignment control," in *17th IFAC World Congress*, Seoul, KR, 2008, pp. 3781–3786.

[5] D. Dirksz, J. Scherpen, and R. Ortega, "Interconnection and damping assignment passivity-based control for Port-Hamiltonian mechanical systems with only position measurements," in *47th IEEE Conference on Decision and Control*, Cancun, MX, 2008, pp. 4957–4962.

[6] M. Jankovic, D. Fontaine, and P. Kokotovic, "TORA example: Cascade and passivity-based control designs," *IEEE Transactions on Control Systems Technology*, vol. 4, no. 3, pp. 292–297, 1996.

[7] R. Olfati-Saber, "Nonlinear control and reduction of underactuted systems with symmetry I: Actuated shape variables case," in *40th IEEE Conference on Decision and Control*, Orlando, FL, US, 2001, pp. 4158–4163.

[8] A. Pavlov, B. Jansen, N. van der Wouw, and H. Nijmeijer, "Experimental output regulation for the TORA system," in *44th IEEE Conference on Decision and Control*, Sevilla, ES, 2005, pp. 1108–1113.

[9] T. Tanighuchi and M. Sugeno, "Piecewise multi-linear model based control for TORA system via feedback linearization," in *International MultiConference of Engineers and Computer Scientist*, Honk Kong, CN, 2018.

[10] k. Tanaka, T. Tanighuchi, and H. Wang, "Model-based fuzzy control of the TORA: Fuzzy regulator and fuzzy observer design via LMIs that rapresent decay rate, disturbance rejection, robustness, optimality," in *1998 IEEE International Conference on Fuzzy Systems Proceedings.*, Anchorage, AK, US, 1998.

[11] L. ChenHung, H. Lin, and H. Chung, "Design of self-tuning fuzzy sliding mode control for tora system," *Expert Systems with Applications*, vol. 32, no. 1, pp. 201–212, 2007.

[12] R. Ortega, M. Spong, F. Gómez-Estern, and G. Blankenstein, "Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment," *IEEE Transactions on Automatic Control*, vol. 47, no. 8, pp. 1218–1233, 2002.

[13] M. Ryalat and D. Laila, "A simplified IDA–PBC design for underactuated mechanical systems with applications," *European Journal of Control*, vol. 27, pp. 1–16, 2016.

[14] R. Olfati-Saber, "Nonlinear control of underactuated mechanical systems with application to robotics and aerospace veichles," Ph.D. dissertation, Massachusetts Institute of Technology, 2001.