Robust energy shaping for mechanical systems with dissipative forces and disturbances

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***Abstract*— This paper presents a novel energy shaping-based integral action for mechanical systems with unknown dissipative forces and matched disturbances. The proposed approach builds on the simultaneous interconnection and damping assignment method and takes advantage of the representation of the dissipative forces in the port-Hamiltonian dynamics. We consider dissipative forces that cannot be written in the classical dissipation structure of the port-Hamiltonian systems. We show that the proposed design ensures the stability of the equilibrium and is robust against dissipative force uncertainty, and rejects constant matched disturbances. Two case studies are presented, and simulation results show the closed-loop performance.**

I. INTRODUCTION

Energy shaping is a well-established passivity-based control (PBC) technique for the stabilisation of mechanical systems. The method's main idea is to use a state-feedback control to shape the energy function of the system and assign a minimum at the desired equilibrium. The closed loop preserves the physical structure of the system, and thus stability can be ensured. The injection of additional damping is in general needed for asymptotic stability (see [1] for a detailed survey on the topic). In many cases, the total energy function of the system needs to be shaped. This task requires solving the so-called kinetic energy matching equation (KE-ME) and the potential energy matching equation (PE-ME), which are a set of partial differential equations (PDE) [2]. A considerable amount of research effort has been dedicated to simplifying or avoiding the task of solving the PDE resulting from the interconnection and damping assignment (IDA) method, see e.g. [3] and [4]. In that direction, the use of gyroscopic forces in the target dynamics introduces an additional degree of freedom that helps to simplify the solution of the KE-ME [5]. A more general class of forces was first proposed in [6] and later considered in [7] and [8]. The dissipative forces allow obtaining a larger class of controllers for mechanical systems. The method in [8], unlike the two-step IDA-PBC approach, performs the energy shaping and damping injection steps together by using dissipative forces in a process called simultaneous interconnection and damping assignment passivity-based control (SIDA-PBC). PBC methods enjoy specific robustness properties as they aim at avoiding cancellation. However, disturbances and friction forces might result in the degradation of the closed-loop performance or instabilities [7], [9], which motivates the development of methods for robust passivity-based control design. In this paper, we present a method to add integral actions (IA) to SIDA-PBC controllers for mechanical systems robust against dissipative forces and matched disturbances.

The objective of adding an IA to the passive output of port-Hamiltonian (pH) systems preserving the interconnection structure was first proposed in [10]. The addition of IA to outputs that are not necessarily passive as an outer loop to enhance the robustness of IDA-PBC controllers against disturbances was proposed in [11], and other fundamental properties discussed in [12]. A nonlinear PID-like controller for underactuated pH mechanical systems was presented in [9], and it was shown to ensure the stability of the desired equilibrium and compensate for disturbances. The previous work utilises a state transformation that preserves the pH form in the new states. An alternate method for IA control design that does not rely on state transformation was proposed in [13]. The method was extended in [14] by relaxing several previously needed assumptions and considering modelling uncertainties in the dynamics.

The main contributions of the paper are as follows.

- We consider IA for mechanical systems with disturbances and implicit dissipative forces, and provide a formal stability proof of the desired equilibrium in the presence of uncertainties. This complement the result in [14], where friction forces in the form $R(q)\dot{q}$ that can be explicitly written in the standard dissipation matrix structure are considered.
- We also show that the IA design can be applied for SIDA-PBC control, where the dissipative forces can be written in implicit form. Previous works considered IA for control system designed using IDA-PBC, where the damping has explicit form ([9]–[11], [14], [15]).

The outline of the paper is as follows. A summary of the SIDA-PBC technique is presented in Section II. The problem formulation and the proposed design are presented in Section III. In Section IV, the control design is applied to two control benchmarks, and the simulation results of the closed loop are shown. Conclusions are presented in Section V.

The research leading to these results has been supported by the PRINBOT project, in the frame of the PRIN 2017 research program, grant number 20172HHNK5_002.

*This work was partially conducted during the visit of the first author to the PRISMA Lab, University of Naples.

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II. Simultaneous Interconnection and Damping Assignment Passivity-Based Control

In this section, we consider a mechanical system described in pH form with friction forces which can be written as

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix} = \begin{bmatrix}O_{n\times n} & I_n\\ -I_n & O_{n\times n}\end{bmatrix}\begin{bmatrix}\partial_q H(q,p)\\ \partial_p H(q,p)\end{bmatrix} + \begin{bmatrix}O_{n\times m}\\ F_r(\dot q)\end{bmatrix} + \begin{bmatrix}O_{n\times m}\\ G(q)\end{bmatrix} u, \tag{1}$$

where $q \in \mathbb{R}^n$ and $p \in \mathbb{R}^n$ are the configuration and momenta vector, respectively; $u \in \mathbb{R}^m$ is the control input vector; $G \in \mathbb{R}^{n\times m}$ is the input matrix with rank m; $O_{n\times n}, I_n \in \mathbb{R}^{n\times n}$ are zero and identity matrices, respectively, of proper dimensions; $F_r(\dot q) \in \mathbb{R}^n$ is the vector of friction forces; $H : \mathbb{R}^{2n} \to \mathbb{R}$ is the total energy function given by

$$H(q,p) = \frac{1}{2} p^T M(q)^{-1} p + V(q); \tag{2}$$

where $M(q) = M^T(q) \in \mathbb{R}^{n\times n}$ is the positive definite inertia matrix; and $V(q) \in \mathbb{R}^n \to \mathbb{R}$ is the potential energy. We also define the column vectors $\partial_q H := \frac{\partial H}{\partial q}$ and $\partial_p H := \frac{\partial H}{\partial p}$

Remark. We consider friction forces described in implicit form, that is the function $F_r(\dot q) = F_r(M^{-1}(q)p)$ cannot be written as $R(q)\dot q$, with $R = R^T \geq 0$, for example Coulomb frictions modelled by $\arctan(\dot q)$.

The problem of stabilising a desired equilibrium point $(q,p) = (q^*, 0_{n\times 1})$ can be solved using SIDA-PBC by finding state-feedback control law such that the closed-loop dynamics can be written as follows

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix} = \begin{bmatrix}O_{n\times n} & M^{-1}(q)M_d(q)\\ -M_d(q)M^{-1}(q) & O_{n\times n}\end{bmatrix} \times \begin{bmatrix}\partial_q H(q,p)\\ \partial_p H(q,p)\end{bmatrix} + \begin{bmatrix}0_n\\ C(q,p)\end{bmatrix}, \tag{3}$$

with the new total energy function given by

$$H_d(q,p) = \frac{1}{2} p^T M_d^{-1}(q) p + V_d(q), \tag{4}$$

where $M_d(q) = M_d^T(q) \in \mathbb{R}^{n\times n}$ is the positive definite desired inertia matrix, $V_d(q) : \mathbb{R}^n \to \mathbb{R}$ is the desired closed-loop potential energy such that $q^* = \arg\ \min V_d(q)$, i.e., V_d has a strict minimum at q^*, and $C(q,p) \in \mathbb{R}^n$ is the vector of desired dissipative forces that satisfies

$$p^T M_d^{-1}(q) C(q,p) \leq 0. \tag{5}$$

The control law that renders the closed loop in the form (3) exists if the following matching equations are satisfied

$$G^{\perp}\{\partial_q(p^T M^{-1} p) - M_d M^{-1} \partial_q(p^T M_d^{-1} p) - 2F_r(\dot q) + 2C(q,p)\} = 0, \tag{6}$$

$$G^{\perp}\{\partial_q V - M_d M^{-1} \partial_q V_d\} = 0. \tag{7}$$

Then, the control law $u = \Psi(q,p)$ that stabilises the desired equilibrium $(q^*, 0_{n\times 1})$ is obtained as

$$\Psi(q,p) = (G^T G)^{-1} G^T \{\partial_q H - M_d M^{-1}(\partial_q H_d + C(q,p) + \partial_q V - M_d M^{-1} \partial_q V_d\}. \tag{8}$$

Moreover, the desired equilibrium is asymptotically stable if the maximum invariant set included in $\mathcal{S} = \{(q,p) \in \mathbb{R}^n \times \mathbb{R}^n | p^T M_d^{-1}(q) C(q,p) = 0\}$ under the dynamics (3) is $(q^*, 0_{n\times 1})$. A possible selection of the desired dissipative forces is [7]

$$C(q,p) = -G K_v(q,p) G^T M_d^{-1} p + F_r(\dot q), \tag{9}$$

where $K_v \in \mathbb{R}^{m\times m}$ is a positive definite symmetric matrix. As done in [7], the matrix K_v can be selected to satisfy (5), even under uncertainty in the friction forces. Moreover, the dissipative force C can satisfy

$$p^T M_d^{-1} C(q,p) \leq -p^T M_d^{-1} R_d M_d^{-1} p \leq 0. \tag{10}$$

for some $R_d > 0$. As discussed in [7], this condition is related to strongly dissipative systems, and it was also considered in [14].

III. SIDA-PBC with Disturbance

In this section, we consider the effect of matched disturbances in the mechanical system (1), which can then be written as

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix} = \begin{bmatrix}O_{n\times n} & I_n\\ -I_n & O_{n\times n}\end{bmatrix}\begin{bmatrix}\partial_q H(q,p)\\ \partial_p H(q,p)\end{bmatrix} + \begin{bmatrix}O_{n\times m}\\ F_r(\dot q)\end{bmatrix} + \begin{bmatrix}O_{n\times m}\\ G(q)\end{bmatrix} (u+d), \tag{11}$$

where $d \in \mathbb{R}^m$ is the disturbance.

The dynamics (11) in closed loop with the controller $u = \Psi(q,p) + \nu$ can be written as

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix} = \begin{bmatrix}O_{n\times n} & M^{-1}(q)M_d(q)\\ -M_d(q)M^{-1}(q) & O_{n\times n}\end{bmatrix}\begin{bmatrix}\partial_q H_d(q,p)\\ \partial_p H_d(q,p)\end{bmatrix} + \begin{bmatrix}0_n\\ C(q,p)\end{bmatrix} + \begin{bmatrix}0_{n\times m}\\ G(q)\end{bmatrix}(\nu + d), \tag{12}$$

where $\nu \in \mathbb{R}^m$ is additional term in the control law that will be used for the outer loop controller to compensate the disturbance.

In general, the action of the disturbance can shift the equilibrium or, even worse, produce instabilities. To prevent that undesirable behaviour, we propose the design of a dynamic controller

$$\begin{aligned}\nu &= \Phi(q,p,\zeta)\\ \dot\zeta &= \Pi(q,p,\zeta)\end{aligned} \tag{13}$$

where $\zeta^* \in \mathbb{R}^m$ is the controller state, $\Phi : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ and $\Pi : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ are smooth functions to be designed such that the dynamics of the system (12) in closed loop with the controller (13) has an asymptotic stable equilibrium at $(q,p,\zeta) = (q^*, 0, \zeta^*)$, for some $\zeta^* \in \mathbb{R}^m$, and a constant disturbance d.

In this paper, we assume that the input matrix has the form $G = \begin{bmatrix} 0_{m\times n-m} & I_m \end{bmatrix}^T$, which simplifies the notation. The transformation proposed in [14] can be used to render the input matrix in the desired form, but the notation becomes complicated and for clarity we made the assumption on G. We also consider uncertainty in the friction force $F_r(\dot{q})$ and follow the approach in [7] so that the matrix K_v is selected to satisfy (10). Note that due to the uncertainty in $F_r(\dot{q})$, the matrix $C(q,p)$ is not perfectly known.

A. Controller design

In general, the model of the friction forces $F_r(\dot{q})$ presents uncertainties and thus the dissipative forces $C(q,p)$ as defined in (9) also present uncertainties. In this section, we present an integral controller that compensates for unknown disturbances and is robust against friction uncertainties.

Since the friction forces are not known exactly, the vector of dissipative forces cannot be computed. Instead, we define the estimated dissipative force vector as follows

$$\hat{C}(q,p) = -GK_vG^TM_d^{-1}p + \hat{F}_r(\dot{q}), \tag{14}$$

where $\hat{F}_r$ is an estimation of the friction forces. We also define the dissipative force error vector as

$$\tilde{C}(q,p) = C(q,p) - \hat{C}(q,p) = F_r(\dot{q}) - \hat{F}_r(\dot{q}) \tag{15}$$

Assumption 1: The vector of friction forces is $F_r(\dot{q}) = [F_{r,1}(\dot{q}_1)\ F_{r,2}(\dot{q}_2)\ \cdots\ F_{r,n}(\dot{q}_n)]^T$ and it satisfies

$$\frac{\beta_{i,min}\dot{q}_i}{\sqrt{\lambda_{i,max}^2 + \dot{q}_i^2}} \le F_{r,i}(\dot{q}_i) \le \frac{\beta_{i,max}\dot{q}_i}{\sqrt{\lambda_{i,min}^2 + \dot{q}_i^2}} \tag{16}$$

where $\lambda_{i,min}, \lambda_{i,max}, \beta_{i,min}, \beta_{i,max} > 0$ and $i = 1, \cdots, n$ representing each coordinate.

This assumption has been considered in [7] and introduces the upper and lower bounds of the friction forces. Then, we define the i-th nominal friction force $\hat{F}_{r,i} = \frac{\hat{\beta}_i}{\sqrt{\hat{\lambda}_i^2 + \dot{q}_i^2}}\dot{q}_i$, where $\hat{\beta}_i$ and $\hat{\lambda}_i$ are the nominal coefficients such that the nominal friction force is within the bounds in (16). As shown in Figure 1, both the actual friction force and the nominal force are within the force bounds. As the functions representing the forces are bounded, we can obtain a bound for the force error as follows

$$\left|\tilde{F}_{r,i}(\dot{q}_i)\right| = \left|F_{r,i}(\dot{q}_i) - \hat{F}_{r,i}(\dot{q}_i)\right| \le \frac{\epsilon_i}{\sqrt{\delta_i^2 + \dot{q}_i^2}} \left|\dot{q}_i\right|, \tag{17}$$

for some $\epsilon_i, \delta_i > 0$, and thus

$$\left|\tilde{F}_{r,i}\right| = \left|F_{r,i} - \hat{F}_{r,i}\right| \le \gamma_i \left|\dot{q}_i\right|, \tag{18}$$

with $\gamma_i = \frac{\epsilon_i}{\delta_i}$.

Remark 3.1: The result in this paper also applies for systems with additional viscous friction forces. Indeed, if the viscous coefficient can be bounded, then we can formulate the nominal forces that satisfy the error bound (18), which will be used later in the paper. In general, we can consider friction forces such that the corresponding nominal force functions satisfy (18).

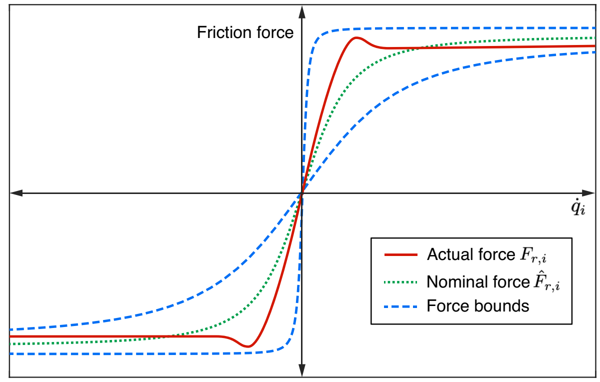


Fig. 1. Friction force, nominal friction force and uncertainty band.

In the following proposition, we present a controller that stabilises the desired equilibrium point, rejects constant disturbances and is robust against uncertainties in the friction forces.

Proposition 3.1: Consider the system (12) in closed loop with the controller

$$\nu = K_1K_I(\zeta - G^Tp), \tag{19}$$

where ζ is the controller state whose dynamics are

$$\dot{\zeta} = -G^TM_dM^{-1}\partial_qH_d - K_1G^TM_d^{-1}p + G^T\hat{C}(q,p), \tag{20}$$

and $K_1, K_I \in \mathbb{R}^{m\times m} > 0$ are controller gains. Then, the closed-loop dynamics can be written in the pH form

$$\begin{bmatrix}\dot{q}\\ \dot{p}\\ \dot{\zeta}\end{bmatrix} = \begin{bmatrix} O_{n\times n} & M^{-1}M_d & M^{-1}M_dG \\ -M_dM^{-1} & 0 & GK_1 \\ -G^TM_dM^{-1} & -K_1G^T & -K_1 \end{bmatrix} \begin{bmatrix}\partial_qW\\ \partial_pW\\ \partial_\zeta W\end{bmatrix} + \begin{bmatrix}0\\ C(q,p)\\ G^T\hat{C}(q,p)\end{bmatrix}, \tag{21}$$

with the closed-loop energy function W defined as

$$W = \frac{1}{2}p^TM_d^{-1}p + \frac{1}{2}(\zeta - \alpha - G^Tp)^TK_I(\zeta - \alpha - G^Tp) + V_d(q), \tag{22}$$

with $\alpha = -K_I^{-1}K_1^{-1}d$, and that has a strict minimum at the desired equilibrium.

Proof: It is sufficient to show that the dynamics (12) and (21) match. We first consider the first row of (12)

$$\begin{aligned}\dot{q} =& M^{-1}p \\ =& M^{-1}M_d\left[M_d^{-1}p - GK_I(\zeta - \alpha - G^Tp)\right] \\ &+ M^{-1}M_dGK_I(\zeta - \alpha - G^Tp) \\ =& M^{-1}M_d\partial_pW + M^{-1}M_dG\partial_\zeta W,\end{aligned}$$

which matches the first row of (21). Following a similar procedure, we use the momentum equation in (12) as follows

$$\begin{aligned}\dot{p} =& -M_dM^{-1}\partial_qH_d + C(q,p) + G\nu + Gd \\ =& -M_dM^{-1}\partial_qH_d + C(q,p) + GK_1K_I(\zeta - G^Tp) \\ & - GK_1K_I\alpha \\ =& -M_dM^{-1}\partial_qH_d + C(q,p) + GK_1K_I(\zeta - G^Tp - \alpha) \\ =& -M_dM^{-1}\partial_qW + GK_1\partial_\zeta W + C(q,p),\end{aligned}$$

which is equivalent to the momentum equation in the closed-loop dynamics (21). Finally, the controller dynamics (20) can also be rewritten as follows

$$\begin{aligned}\dot{\zeta} =& -G^T M_d M^{-1}\partial_q H_d - K_1 G^T M_d^{-1} p + G^T \hat{C}(q,p)\\ =& -G^T M_d M^{-1}\partial_q H_d - K_1 G^T M_d^{-1} p + G^T \hat{C}(q,p)\\ &+ K_1 K_I(\zeta - \alpha - G^T p) - K_1 K_I(\zeta - \alpha - G^T p)\\ =& -G^T M_d M^{-1}\partial_q H_d - K_1 G^T \big[M_d^{-1} p - G K_I\\ &\times (\zeta - \alpha - G^T p)\big] - K_1 K_I(\zeta - \alpha - G^T p)\\ &+ G^T \hat{C}(q,p)\\ =& -G^T M_d M^{-1}\partial_q W - K_1 G^T \partial_p W - K_1 \partial_\zeta W\\ &+ G^T \hat{C}(q,p),\end{aligned}$$

which matches the last row of (21). The matching of the equations shows that the closed-loop dynamics can be written in the form (21) as we wanted to prove. ■

Notice that the point $(q, p, \zeta) = (q^*, 0, \zeta^*)$, with $\zeta^* = -K_I^{-1} K_1^{-1} d$, is an equilibrium of the closed loop (21). Moreover, as we will show in the next section, this equilibrium is asymptotically stable.

B. Closed-loop stability

In this section, we analyse the stability properties of the equilibrium $(q^*, 0, \zeta^*)$ of the closed loop (21). To achieve that objective, we consider the energy function (22) as a Lyapunov candidate and we compute the derivative of W with respect to time along the trajectory solutions of (21). We use the properties of the pH form of the closed loop and the fact that $\partial_p W = M_d^{-1} p - G\partial_\zeta W$. Then, the time derivative of W can be computed as follows

$$\begin{aligned}\dot{W} =& \left[M_d^{-1} p - G K_I(\zeta - \alpha - G^T p)\right]^T \dot{p}\\ &+ (\zeta - \alpha - G^T p)^T K_I \dot{\zeta} + \partial_q^T V_d\, \dot{q}\\ =& -\partial_\zeta^T W K_1 \partial_\zeta W + \partial_p^T W C(q,p) + \partial_\zeta^T W G^T \hat{C}(q,p)\\ =& -\partial_\zeta^T W K_1 \partial_\zeta W + p^T M_d^{-1} C(q,p)\\ &- \partial_\zeta^T W G^T \left[C(q,p) - \hat{C}(q,p)\right]\\ \leq& -\partial_\zeta^T W K_1 \partial_\zeta W - p^T M_d^{-1} R_d M_d^{-1} p\\ &- \partial_\zeta^T W G^T \tilde{C}(q,p)\\ \leq& -\begin{bmatrix} M_d^{-1} p \\ \partial_\zeta W \end{bmatrix}^T \begin{bmatrix} R_d & 0 \\ 0 & K_1 \end{bmatrix} \begin{bmatrix} G^T M_d^{-1} p \\ \partial_\zeta W \end{bmatrix}\\ &- \partial_\zeta^T W G^T \tilde{C}(q,p)\end{aligned} \tag{23}$$

When the friction forces are exactly know, then

$$\dot{W} \leq -\left\|M_d^{-1} p\right\|_{R_d} - \|\partial_\zeta W\|_{K_1} \leq 0, \tag{24}$$

which ensures stability of the equilibrium. Asymptotic stability can be readily shown using invariance principle arguments [16]. In the presence of friction force uncertainty, we can use (18) to bound the friction forces error $\tilde{C}$ in (23) as follows

$$\begin{aligned}\dot{W} \leq& -\begin{bmatrix} M_d^{-1} p \\ \partial_\zeta W \end{bmatrix}^T \begin{bmatrix} R_d & 0 \\ 0 & K_1 \end{bmatrix} \begin{bmatrix} M_d^{-1} p \\ \partial_\zeta W \end{bmatrix} + \partial_\zeta^T W G^T \Gamma \dot{q}\\ \leq& -\begin{bmatrix} M_d^{-1} p \\ \partial_\zeta W \end{bmatrix}^T \begin{bmatrix} R_d & 0 \\ 0 & K_1 \end{bmatrix} \begin{bmatrix} M_d^{-1} p \\ \partial_\zeta W \end{bmatrix}\\ &+ \partial_\zeta^T W G^T \Gamma M^{-1} M_d M_d^{-1} p\\ \leq& -\begin{bmatrix} M_d^{-1} p \\ \partial_\zeta W \end{bmatrix}^T \begin{bmatrix} R_d & \Phi \\ \Phi^T & K_1 \end{bmatrix} \begin{bmatrix} M_d^{-1} p \\ \partial_\zeta W \end{bmatrix}\end{aligned} \tag{25}$$

where $\Phi = \frac{1}{2} M_d M^{-1} \Gamma^T G$ and $\Gamma = \mathrm{diag}(\gamma_1, \cdots, \gamma_n)$, with γ_i defined in (18). Using Schur's lemma and noting that $R_d > 0$, we can select the gain K_1 that should satisfy

$$K_1 - \Phi^T R_d^{-1} \Phi > 0$$

and ensure stability of the equilibrium. Asymptotic stability of the equilibrium follows directly using invariance principle arguments [16].

IV. Case Studies

In this section we apply the proposed control design to two benchmarks, namely the ball and beam and the two-links manipulator. The former is an underactuated mechanical system while the later is fully actuated. The performance of the controller is evaluated in simulations.

A. The ball and beam

The ball and beam system is shown in Fig. 2. The main dynamics of the system can be written in form (11) with input matrix $G = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$, inertia matrix

$$M(q) = \mathrm{diag}(1, L^2 + q_1^2), \;\; G = e_1,$$

and the potential energy

$$V(q) = g q_1 \sin q_2,$$

where q_1 is the position of the ball, q_2 is the angle of the beam, and L is the length of the beam [2]. In addition, we consider the effect of Coulomb friction forces

$$F_{r,i} = \beta_i \arctan \dot{q}_i \tag{26}$$

with $i = 1, 2$.

Following the SIDA design in [7], we can obtain a dynamics in the form (12) with the desired mass matrix

$$M_d(q) = \begin{bmatrix} \sqrt{2}(L^2 + q_1^2)^{-\frac{1}{2}} & 1 \\ 1 & \sqrt{2}(L^2 + q_1^2)^{\frac{1}{2}} \end{bmatrix},$$

and the desired potential energy

$$V_d = g(1 - \cos q_2) + K_P \left[q_2 - \frac{1}{\sqrt{2}} \mathrm{arcsinh}\left(\frac{q_1}{L}\right)\right]^2,$$

where K_P is a positive constant to assign the minimum at the desired equilibrium.

The nonlinear damping injection takes the form (see the design details of the SIDA controller in [7])

$$u_{di} = -K_v \frac{1}{L^2 + q_1^2} \left[-p_1 + \left(\frac{2}{L^2 + q_1^2}\right)^{\frac{1}{2}} p_2\right], \tag{27}$$

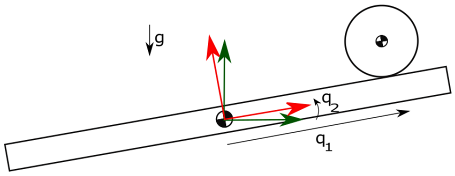


Fig. 2. A schematic of the ball and beam system where q_1 is the position of the ball and q_2 is the deviation of the beam from the fixed world frame represented in green.

where

$$K_v(q,p) > \frac{(L^2+q_1^2)^2}{(p_1\sqrt{L^2+q_1^2}-\sqrt{2}p_2)^2} \times \Big[\beta_{1max}\arctan(\dot{q}_1)\big(p_1\sqrt{2L^2+q_1^2}-p_2\big) + \beta_{2max}\arctan(\dot{q}_2)\big(\frac{\sqrt{2}p_2}{\sqrt{L^2+q_1^2}}-p_1\big)\Big]. \quad (28)$$

To implement the integral action controller (19),(20), we consider a nominal friction coefficient $\hat{\beta}_i \in (\beta_{min}, \beta_{max})$, so that (16) is satisfied. The goal is to stabilise the system at $q^* = (0,0)$. The length of the beam is 1m and the friction coefficients are $\beta_1 = \beta_2 = 10$, $\hat{\beta}_1 = \hat{\beta}_2 = 9$. The controller gains are $K_P = 10$, $K_v(q,p)$ is selected as in [7] , $K_1 = 23.5$ and $K_I = 0.1$. The initial conditions for the simulations are $q_1(0) = 0.5$ m, $\dot{q}_1(0) = 0.5$ m/s, $q_2(0) = 0.1$ rad and $\dot{q}_2 = 0$ rad/s. For comparison purposes, we consider the ball and beam in closed loop with the standard SIDA controller and the proposed SIDA with integral action. Also, a torque disturbance on the beam of 20 N is used in the simulations. Figure 3 shows the time history of the ball position and beam angle. The scenario shows a very good performance of the SIDA controller when there are no disturbances. However, the closed-loop performance deteriorates significantly under the presence of the disturbance and the ball might fall from the beam as shown in Figure 3-(a). On the other side, the SIDA-IA controller can stabilise the desired ball position and the beam angle, and reject the disturbance as shown in Figure 3-(b). The velocity of the ball and angular velocity of the beam converge to zero in all scenarios as shown in Figure 4. The state of the controller and the control torque commanded by the SIDA-IA are shown in Figure 5. Notice that the control input compensates for the disturbance, and the controller state reaches $\zeta^* = -K_I^{-1}K_1^{-1}d$ as expected.

B. The two link manipulator

As a second example, we consider a fully actuated planar elbow manipulator shown in Figure 6. The dynamics model can be written in the form (11) with inertia matrix

$$M(q) = \begin{bmatrix} a_1 + a_2 + 2b\cos(q_2) & a_2 + a_3\cos q_2 \\ a_2 + a_3\cos q_2 & a_2, \end{bmatrix}$$

where a_1, a_2 and a_3 are constant parameters of the model and $V(q) = 0$ (see [17] for details of the model). We also consider friction forces $F_{r,i} = \beta_i \arctan(\dot{q}_i)$, with $i = 1, 2$, and link angles q_1 and q_2. Since the manipulator is fully actuated, then $G = I_2$. The goal is to stabilise a desired pose,

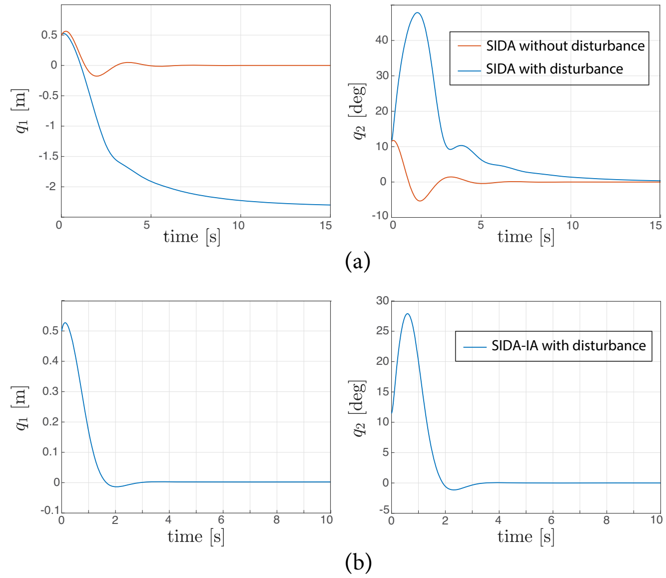


Fig. 3. Time histories of the ball position and beam angle using (a) standard SIDA controller and (b) the proposed SIDA controller with IA.

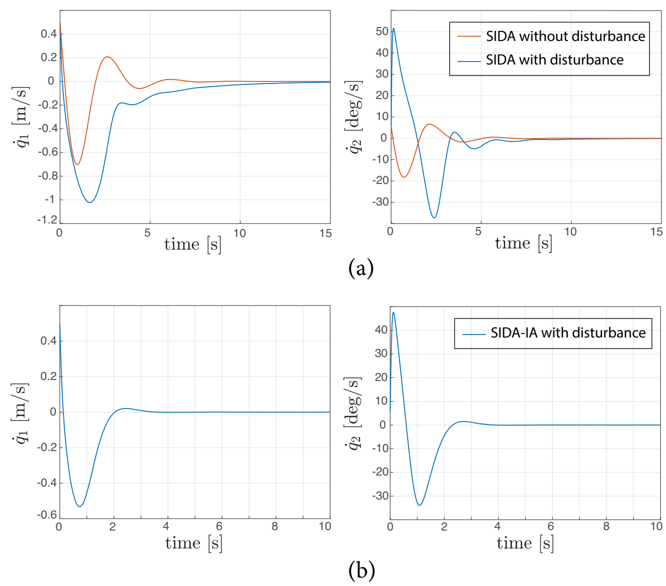


Fig. 4. Time histories of the ball velocity and beam angular velocity using (a) standard SIDA controller and (b) the proposed SIDA controller with IA.

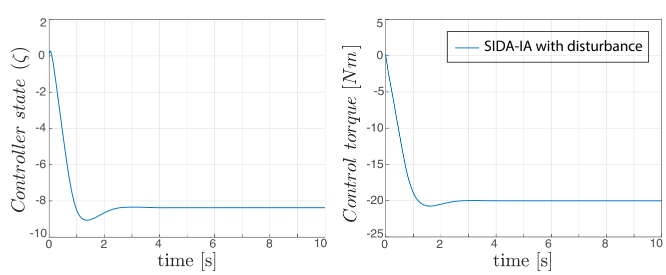


Fig. 5. Time histories of the controller state and control input of the SIDA controller with IA.

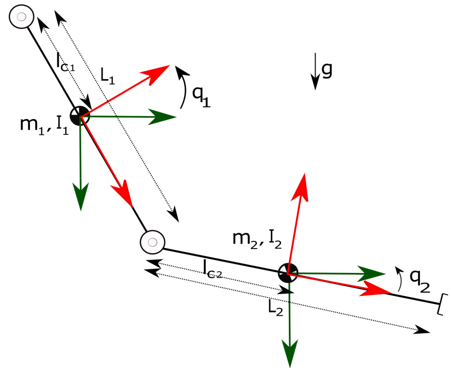
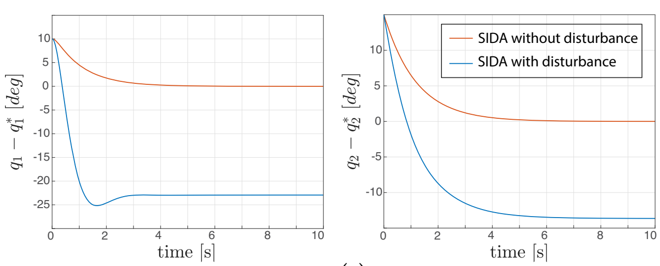


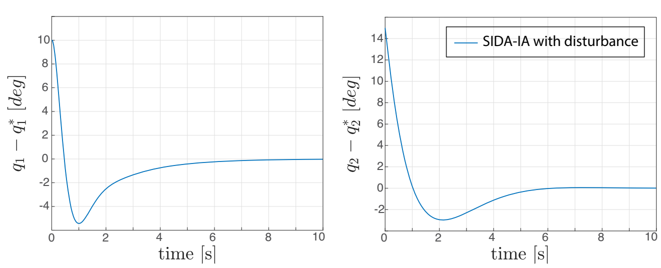
Fig. 6. A schematic of the two link manipulator. q_1 and q_2 are the deviation of the first and the second link from the fixed world frame represented in green, respectively.

which in this case is $q^* = (10, 15)$ deg. We simulate the manipulator in closed loop with the standard SIDA controller and the proposed SIDA controller with IA (19),(20). We use the model parameters in [17] and the friction coefficients $\beta_1 = \beta_2 = 0.02$, $\hat{\beta}_1 = \hat{\beta}_2 = 0.01$. The controller gains are $K_1 = \text{diag}(1.51, 1.51)$ and $K_I = \text{diag}(1.45, 1.45)$.

Figure 7 shows the time history of the link positions errors. The SIDA controller performs well without disturbances, but it suffers from significant steady-state error under the action of disturbances and friction uncertainty. The proposed SIDA controller with integral action can achieve the set point without steady-state error and acceptable transient response, which shows the robustness properties of the closed loop against friction uncertainty and disturbances.



(a)



(b)

Fig. 7. Time histories of link position errors using (a) standard SIDA controller and (b) the proposed SIDA controller with IA.

V. CONCLUSIONS

This paper presented an integral action controller for mechanical systems with matched constant disturbances and dissipative forces represented in implicit pH form. We show that the controller is robust against uncertainty in the dissipative forces and compensate for constant disturbances. The proposed control design was applied to the ball and beam and a two-link manipulator. The case studies show that the IA design can be applied to fully actuated and underactuated systems. The simulations show the controller performance and its robustness and disturbance rejection properties.

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